Induced gravity inflation in the standard model of particle physics

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Abstract

We are considering the cosmological consequences of an induced gravity theory coupled to the minimal standard model of particle physics. The non-minimal coupling parameter between gravity and the Higgs field must then be very large, yielding some new cosmological consequences for the early Universe and new constraints on the Higgs mass. As an outcome, new inflation is only possible for very special initial conditions producing first a short contraction era after which an inflationary expansion automatically follows; a chaotic inflationary scenario is successfully achieved. The contrast of density perturbations required to explain the seed of astronomic structures are obtained for very large values of the Higgs mass ($M_H >> G_F^{-1/2}$), otherwise the perturbations have a small amplitude; in any case, the spectral index of scalar perturbations agrees with the observed one.

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1 Introduction

It is well known that inflation is a necessary constituent of modern cosmology in order to solve the long standing problems of standard cosmology [1] as the homogeneity and isotropy of the Universe, the horizon and flatness problems, for reviews see [2, 3, 4, 5]. Indeed, inflation is desired to be accomplished by particle physics theories, which should be able to fulfill the cosmological standard tests [6], before one can speak about a successfully cosmology. However, there are some unwanted problems inherent to many of the inflation models like the achievement of the right contrast of density perturbations within a non fine-tuning particle physics model or enough reheat temperature after inflation to yield the particle content of the Universe, among others; in this sense, one can also speak of some "quasi-long" standing problems of inflationary models.

In order to understand these issues both of cosmology and particle physics, in recent years it has been carried out some progress in the experiments, like the observations made by the Cosmic Background Explorer (COBE) satellite [7, 8], the possible discovery of the top quark [9], among others, as well as some theoretical extensions of the standard model, grand unification theories (GUT)'s and gravity theories, see [2, 3]. Particularly, the cosmological consequences of induced gravity models are well known [10, 11, 12, 13, 14, 15, 16, 17], but the particle physics content is still unclear, simply because the Lagrangians used there imply scalar field associated particles with masses greater than the Planck mass (M_{Pl}). Motivated by these requests we have recently studied [18] an induced gravity model of inflation based on a non-minimal coupling between gravity and the SU(5) GUT Higgs field, as an extension to both the minimal SU(5) GUT and Einstein's General Relativity (GR). Now we study, what is the role played by the induced gravity model with a non-minimal coupling in the standard model of particle physics, motivated by some aspects listed below.

The issue of inflation at the electroweak energy scale has been recently discussed [19] motived by the possibility that baryogenesis could take place at that scale and also because any net baryon number generated before would be brought to zero by the efficient anomalous electroweak processes, unless the original GUT model was B-L conserving [20]. Moreover, inflation in this energy scale brings other advantages: one does not have to deal with heavy relics or monopoles, which usually appear at GUT scales and reheating is more efficient. One can also expect that the some of physics of this scenario could de tested in particle accelerators in the followings years.

Furthermore, recently it has been proposed induced gravity in the standard model could solve some other problems of particle physics and cosmology, like the necessity of the Higgs mass to be order of the theory cut off [21] and to relate directly Mach ideas with a particle production mechanism by means of the equivalence principle, as well as, the missing mass problem [22].

In this paper, we want to discuss some early Universe consequences of a theory of gravity coupled to the isovectorial Higgs field of the standard model, which is proved to produce a type of Yukawa gravitational interaction [23, 24, 25], and which breaks down to give rise to both some boson and fermion masses and the Newton's constant. Since the symmetry-breaking process of the SU(3)×SU(2)×U(1) standard model is expected to occur in the physical Universe, we are considering inflation there.

This paper is organized as follows: in section 2 we introduce the theory of induced gravity in the standard model. The cosmological equations for an isotropic Universe as

well as the slow rollover conditions for inflation are presented in section 3. After that, in section 4 we analyze the two possible scenarios, i.e., a modified version of new inflation as well as a chaotic inflation model, depending essentially on the initial conditions for the Higgs field at the beginning of time. Finally, in section 5 we stress our conclusions.

2 Induced gravity in the standard model

We consider here an induced gravity theory coupled to the minimal standard model of the internal gauge group SU(3)×SU(2)×U(1) with the SU(2)×U(1) Higgs field ϕ. The Lagrange density [22] with units $\hbar = c = k_B = 1$ and the signature (+,-,-,-) is:

$$\mathcal{L} = \left[\frac{\alpha}{16\pi}\phi^\dagger\phi \;\; R + \frac{1}{2}\phi^\dagger_{||\mu}\phi^{||\mu} - V(\phi^\dagger\phi) + L_M\right]\sqrt{-g} \tag{1}$$

where R is the Ricci scalar, the symbol $||\mu$ means in the following the covariant derivative with respect to all gauged groups and represents in (1) the covariant gauge derivative: $\phi_{||} = \phi_{|\mu} + ig[A_\mu, \phi]$ where $A_\mu = A_\mu{}^a\tau_a$ are the gauge fields of the inner symmetry group, τ_a are its generators and g is the coupling constant of the gauge group ($|\mu$ means usual partial derivative); α is a dimensionless parameter to regulate the strenght of gravitation and V is the Higgs potential; L_M contains the fermionic and massless bosonic fields of the standard model (L, R mean summation of left-, right-handed terms):

$$L_M = \frac{i}{2}\bar{\psi}\gamma^\mu_{L,R}\psi_{||\mu} + h.c. - \frac{1}{16\pi}F^a{}_{\mu\nu}F_a{}^{\mu\nu} - k\bar{\psi}_R\phi^\dagger\hat{x}\psi_L + h.c. \tag{2}$$

where ψ summarizes the leptonic and hadronic Dirac wave functions, $F_{\mu\nu a}$ are the gauge-field strengths, $\hat{x}$ represents the Yukawa coupling matrix for the fermionic masses and k its (dimensionless real) coupling constant.

Naturally from the first term of Eq. (1) it follows that $\alpha\ \phi^\dagger\phi$ plays the role of a variable reciprocal gravitational "constant". The aim of our theory is to obtain GR as a final effect of a symmetry breaking process and in that way to have Newton's gravitational constant G induced by the Higgs field; similar theories have been considered to explain Newton's gravitational constant in the context of a spontaneous symmetry-breaking process to unify gravity with other fields involved in matter interactions, but in much higher energy scales, see Refs. [26, 27, 28, 18].

In this paper we want to stress the cosmological consequences when the symmetry-breaking of the SU(2)×U(1) Higgs field is responsible for the generation of gravitational constant as well as the electroweak standard particle content, i.e., all the fermionic masses as well as the $W^\pm$ -, Z-boson masses.

The Higgs potential takes the form,

$$V(\phi) = \frac{\lambda}{24}\left(\phi^2 + 6\frac{\mu^2}{\lambda}\right)^2 \tag{3}$$

where we added a constant term to prevent a negative cosmological constant after the breaking. The Higgs ground state, v, is given by

$$v^2 \;\; = \;\; -\frac{6\mu^2}{\lambda} \tag{4}$$

with $V(v) = 0$, where λ is a dimensionless real constant, whereas μ^2 (< 0) is so far the only dimensional real constant of the Lagrangian.

In such a theory, the potential $V(\phi)$ will play the role of a cosmological "function" (instead of a constant) during the period in which ϕ goes from its initial value ϕ_o to its ground state, v, where furthermore

$$G \;=\; \frac{1}{\alpha v^2} \tag{5}$$

is the gravitational constant to realize from (1) the theory of GR [22]. In this way, Newton's gravitational constant is related in a natural form to the mass of the gauge bosons, which for the case of the standard model one has

$$M_W \;=\; \sqrt{\pi}\mathrm{g}v \quad . \tag{6}$$

As a consequence of (5) and (6) one has that the strength parameter for gravity, α , is determinated by

$$\alpha \;=\; 2\pi\left(\mathrm{g}\frac{M_{Pl}}{M_W}\right)^2 \;\approx\; 10^{33} \tag{7}$$

where $M_{Pl} = 1/\sqrt{2G}$ is the Planck mass and $\mathrm{g} \approx 0.18$. In this way, the coupling between the Higgs field and gravitation is very strong: the fact that $\alpha >> 1$ is the price paid in recovering Newton's gravitational constant at that energy scale and it brings some important differences when compared to the standard induced gravity models [18], where to achieve successful inflation typically $\alpha << 1$ [11, 13], and in that way, the existence of a very massive particle ($> M_{Pl}$) is necessary, which after inflation should decay into gravitons making difficult later an acceptable nucleosynthesis scenario [29].

From (1) one calculates immediately the gravity equations of the theory

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \frac{8\pi V(\phi^\dagger\phi)}{\alpha\ \phi^\dagger\phi}g_{\mu\nu} \;&=\; \\ -\frac{8\pi}{\alpha\ \phi^\dagger\phi}T_{\mu\nu} - \frac{8\pi}{\alpha\ \phi^\dagger\phi}&\left[\phi^\dagger_{(||\mu}\phi_{||\nu)} - \frac{1}{2}\phi^\dagger_{||\lambda}\phi^{||\lambda}\ g_{\mu\nu}\right] \\ -\frac{1}{\phi^\dagger\phi}&\left[(\phi^\dagger\phi)_{|\mu||\nu} - (\phi^\dagger\phi)^{|\lambda}{}_{||\lambda}\ g_{\mu\nu}\right] \end{aligned} \tag{8}$$

where $T_{\mu\nu}$ is the energy-momentum tensor belonging to Eq. (2), and the Higgs field equations are

$$\phi^{||\lambda}{}_{||\lambda} + \frac{\delta V}{\delta\phi^\dagger} - \frac{\alpha}{8\pi}R\phi \;=\; 2\frac{\delta L_M}{\delta\phi^\dagger} \;=\; -2k\bar{\psi}_R\hat{x}\psi_L \quad . \tag{9}$$

In the unitary gauge the Higgs field ϕ takes the form, avoiding Golstone bosons,

$$\phi \;=\; v\sqrt{1+2\chi}N \ , \ \ \phi^\dagger\phi = v^2(1+2\chi)N^\dagger N = v^2(1+2\chi) \ , \quad N = const. \tag{10}$$

where the new real scalar variable χ describes the excited Higgs field around its ground state; for instance $\phi = 0$ implies $\chi = -1/2$ and $\phi = vN$ implies $\chi = 0$. With this new Higgs variable Eqs. (8) and (9) are now:

$$
\begin{aligned}
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \left[\frac{8\pi}{\alpha v^2} \frac{V(\chi)}{(1+2\chi)} \right] g_{\mu\nu} \;\; &= \\
-\frac{8\pi}{\alpha v^2} \frac{1}{(1+2\chi)} \hat{T}_{\mu\nu} - \frac{8\pi}{\alpha} \frac{1}{(1+2\chi)^2} & \left[\chi_{|\mu} \chi_{|\nu} - \frac{1}{2} \chi_{|\lambda} \chi^{|\lambda} g_{\mu\nu} \right] \\
-\frac{2}{1+2\chi} \left[\chi_{|\mu||\nu} - \chi^{|\lambda}{}_{||\lambda} g_{\mu\nu} \right] & \qquad (11)
\end{aligned}
$$

where $\hat{T}_{\mu\nu}$ is the *effective* energy-momentum tensor given by

$$
\hat{T}_{\mu\nu} \;=\; T_{\mu\nu} + \frac{(1+2\chi)}{4\pi} \, M^2_{ab} \left(A^a{}_\mu A^b_\nu - \frac{1}{2} g_{\mu\nu} A^a{}_\lambda A^{b\lambda} \right), \qquad (12)
$$

with $M^2_{ab} = 4\pi g^2 v^2 N^\dagger \tau_{(a} \tau_{b)} N$ the gauge boson mass square matrix.

The scalar field equation becomes, after an automatically cancellation of the gauge boson matter terms, a homogeneous Klein-Gordon equation,

$$
\chi^{|\mu}{}_{||\mu} + \frac{1}{(1+\frac{4\pi}{3\alpha})} \frac{4\pi}{9\alpha} \lambda v^2 \, \chi \;\equiv\; 0 \qquad (13)
$$

from which one can read immediately the mass of the Higgs boson M_H , and therefore its Compton range l_H,

$$
M_H \;=\; \sqrt{\frac{\frac{4\pi}{9\alpha} \lambda v^2}{(1+\frac{4\pi}{3\alpha})}} \;\;, \;\; l_H \;=\; \frac{1}{M_H} \;\;, \qquad (14)
$$

whereby one has that the Higgs particle mass is a factor $\sqrt{\frac{4\pi}{3\alpha}} \approx 10^{-17}$ smaller than the one derived from the standard model without gravitation, for an anternative derivation see Ref. [21]. This is a very interesting property since the Higgs mass determines the scale of the symmetry-breaking and, moreover, $\sqrt{\lambda}/\alpha$ shall be a very small quantity that determines the magnitud of the density perturbations (see later discussion).

It is worth noting that Eq. (13) has no source: the positive trace T contribution to the source turns out to be equal in magnitude to the negative fermionic contribution, in such a way that they cancel each other exactly [22]. Then, no only the gauge bosonic but also the fermionic masses no longer appear in this equation as a source of the excited Higgs field, which is just coupled to the very weak gravitational field contained in the only space-time covariant derivative. For this reason, once the symmetry breaks down at the early Universe the Higgs particle remains decoupled from the rest of the world, interacting merely by means of gravity. Therefore, it is virtually impossible to generate the Higgs field χ or its associated particle in the laboratory. Consequently, its current experimental below mass limit of $64 GeV$ does not necessarily apply here. In fact, we shall see that in order to achieve successful cosmology, its value could be much greater than this.

The energy-momentum conservation law reads

$$\hat{T}_{\mu}{}^{\nu}{}_{||\nu} = \frac{\chi_{|\mu}}{1+2\chi}\left[\sqrt{1+2\chi}\ \bar{\psi}\hat{m}\psi - \frac{1+2\chi}{4\pi}M_{ab}^2 A^a{}_{\lambda}A^{b\lambda}\right] = \frac{\chi_{|\mu}}{1+2\chi}\hat{T}, \qquad (15)$$

where $\hat{m} = 1/2\ kv(N^{\dagger}\hat{x} + \hat{x}^{\dagger}N)$ is the fermionic mass matrix, see Ref. [22]. The source of this equation is partially conterbalenced by the fermionic and bosonic matter fields, whose masses are acquired at the symmetry-breaking.

The potential term, which shall play the role of a positive cosmological function (see the square brackets on the left hand hand of Eq. (11)), takes in terms of χ the simple form,

$$V(\chi) \;=\; \frac{\lambda v^4}{6}\chi^2 \;=\; (1+\tfrac{4\pi}{3\alpha})\frac{3}{8\pi G}M_H^2\chi^2 \qquad (16)$$

which at the ground state vanish, $V(\chi=0)=0$. Note that $V(\chi) \sim M_{Pl}^2 M_H^2 \chi^2$; this fact is due to the relationship (5) to obtain GR once the symmetry-breaking takes place. Then, from Eq. (11) and (5) one recovers GR for the ground state

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \;=\; -8\pi G\ \hat{T}_{\mu\nu} \qquad (17)$$

with the effective energy-momentum tensor, $\hat{T}_{\mu\nu}$, given by Eq. (12). Newton's gravitational "function" is $G(\chi) = \frac{1}{\alpha v^2}\frac{1}{1+2\chi}$ and Newton's gravitational constant $G(\chi = 0) = G$.

Next, we proceed to investigate the cosmological consequences of such a theory.

3 FRW- MODELS

Let us consider a Friedman-Robertson-Walker (FRW) metric. One has with the use of (5) that Eqs. (11) are reduced to

$$\frac{\dot{a}^2+\epsilon}{a^2} \;=\; \frac{1}{1+2\chi}\left(\frac{8\pi G}{3}[\rho + V(\chi)] - 2\frac{\dot{a}}{a}\dot{\chi} + \frac{4\pi}{3\alpha}\frac{\dot{\chi}^2}{1+2\chi}\right) \qquad (18)$$

and

$$\frac{\ddot{a}}{a} = \frac{1}{1+2\chi}\left(\frac{4\pi G}{3}[-\rho - 3p + 2V(\chi)] - \ddot{\chi} - \frac{\dot{a}}{a}\dot{\chi} - \frac{8\pi}{3\alpha}\frac{\dot{\chi}^2}{1+2\chi}\right)\ , \qquad (19)$$

where $a = a(t)$ is the scale factor, ϵ the curvature constant ($\epsilon = 0,\ +1\ or\ -1$ for a flat, closed or open space, correspondingly), ρ and p are the matter density and pressure assuming that the effective energy momentum tensor (12) has in the classical limit the structure of that of a perfect fluid. An overdot stands for a time derivative.

In the same way Eq. (13) results in:

$$\ddot{\chi} + 3\frac{\dot{a}}{a}\dot{\chi} + M_H^2\chi \;=\; 0\ . \qquad (20)$$

The Higgs mass demarcates the time epoch for the rolling over of the potential, and therefore for inflation.

The continuity Eq. (15) is

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) \quad = \quad \frac{\dot{\chi}}{1+2\chi} \;\; (\rho - 3p). \tag{21}$$

The matter density decreases as the Universe expands, but increases by the mass production due to the Higgs mechanism. If one takes the equation of state of a barotropic fluid, i.e. $p = \nu\rho$ with the dimensionless constant ν, Eq. (21) can be easily integrated

$$\rho \;=\; \frac{M}{a^{3(1+\nu)}}(1+2\chi)^{\frac{1}{2}(1-3\nu)} = \begin{cases} M(1+2\chi)^2 & if \quad \nu = -1 \;\; (anti-stiff\ matter) \\ \frac{M}{a^3}(1+2\chi)^{\frac{1}{2}} & if \quad \nu = 0 \quad\; (dust) \\ \frac{M}{a^4} & if \quad \nu = 1/3 \;\; (radiation), \end{cases} \tag{22}$$

where M is the integration constant. For the dust and "anti-stiff" matter models there is an extra factor, because of the production mechanism; if the Higgs field is at the beginning of time very near to its metastable equilibrium state, $\chi \sim -1/2$, there is neither beginning mass for the Universe nor size, see Eq. (25). For the radiation case Eq. (21) is sourceless; then there is no entropy production allowed by the Higgs mechanism: a radiation fluid in this theory acts as a decoupling agent between matter and Higgs field. For $\nu = 0$ the mass of the Universe $M(\chi) \approx \rho a^3 = M(1+2\chi)^{1/2}$ increases from zero to a final value M, if the initial Higgs value is [1] $\chi_o \approx -1/2$ (new "inflation"); on the other side for $\chi_o > 1$ (chaotic inflation) the mass decreases a factor $(1+2\chi_o)^{1/2} < 10$ (as will become clear later), i.e., the today observed baryonic mass should be given by M if there were not extra matter production after inflation. One notes that the presence of the different types of matter densities (relativistic or dust-like particles) is relevant for the physical processes that take place, i.e., entropy production processes, also when they bring no important dynamical effects if the inflation potential dominates.

One notes that the Higgs potential is indeed a positive cosmological function, which corresponds to a positive mass density and a negative pressure (see Eqs. (18) and (19)), and represents an ideal ingredient to have inflation; that is, $V(\chi)$ shall be the "inflaton" potential. But, on the other hand, there is a negative contribution to the acceleration Eq. (19) due to the Higgs-kinematic terms, i.e., terms involving $\dot{\chi}$ and $\ddot{\chi}$; terms involving the factor $1/\alpha \sim 10^{-33}$ are simply too small compared to the others and can be neglected.

For inflation it is usually taken that $\ddot{\chi} \approx 0$, but in fact the dynamics should show up this behavior or at least certain consistency. For instance, in GR with the *ad hoc* inclusion of a scalar field ϕ as a source for the inflation, one has that at the "slow rollover" epoch $\ddot{\phi} \approx 0$ and therefore $\dot{\phi} = -V'/3H$, which implies that

$$\frac{\ddot{\phi}}{3H\dot{\phi}} = -\frac{V''}{9H^2} + \frac{1}{48\pi G}\left(\frac{V'}{V}\right)^2 << 1 \quad , \tag{23}$$

where $H = \dot{a}/a$ is the Hubble expansion rate (a prime denotes the derivative with respect to the corresponding scalar field, see Ref. [6]). In the present theory, if one

[1] The subindex "o" stands for the initial values (at $t = 0$) of the corresponding variables.

considers the Higgs potential term in Eq. (18) as the dominant one [2] and Eq. (20) without source, i.e., $p = \frac{1}{3}\rho$, one has indeed an extra term due to the variation of Newton's "function" $G(\chi)$, that is

$$\frac{\ddot{\chi}}{3H\dot{\chi}} = \left(\frac{1}{1+\frac{4\pi}{3\alpha}}\right)\frac{4\pi G}{3}\left[-\frac{V''}{9H^2} + \frac{1}{48\pi G}\left(\frac{V'}{V}\right)^2(1+2\chi) - \frac{1}{24\pi G}\frac{V'}{V}\right] \quad . \tag{24}$$

If $\chi < 0$ the last term does not approach to zero during the rolling down process for $\alpha >> 1$. Thus, one has instead of a "slow", rather a "fast" rollover dynamics of the Higgs field along its potential down hill. On the other side, for $\chi >> 1$ there is indeed a "slow" rollover dynamics.

With this in mind one has to look carefully at the contribution of $\ddot{\chi}$: if one brings $\ddot{\chi}$ from (20) into (19) one has that $M_H^2\chi$ competes with the potential term $M_H^2\chi^2$, and during the rolling down of the potential, when χ goes from $-1/2$ to 0, $M_H^2\chi < 0$ dominates the dynamics, and therefore instead of inflation one ends with deflation or at least with a contraction era for the scale factor; how strong is the contraction era, should be determined by the set of initial conditions $(\chi_o, \dot{\chi}_o)$.

Resuming, if one starts the Universe evolution with an ordinary new inflation scenario ($\chi_o < 0$), it implies in this theory a "short" deflation instead of a "long" inflation period, since the Higgs field goes relatively fast to its minimum. This feature should be present in theories of induced gravity with $\alpha > 1$ and also for the BDT with this type of potential (see for example the field equations in Ref. [30]). Considering the opposite limit, $\alpha < 1$, induced gravity models [11] have proven to be successful for inflation, also if one includes other fields [15]; induced gravity theories with a Coleman-Weinberg potential are also shown to be treatable for a very small coupling constant λ with $\chi_o < 0$ [13, 14], or with $\chi_o > 0$ [12] and $\alpha < 1$ as well as $\alpha > 1$ [10, 16, 17]. For extended or hyperextended inflation models [31, 32] this problem does not arise because of the presence of vacuum energy during the rollover stage of evolution, which is supposed to be greater than the normal scalar field contribution.

With this concern one has to prepare a convenient scenario for the Universe to begin with. But first we would like to mention that there are some important aspects to be considered in the theory of the electroweak phase transition in order to realize a more realistic cosmological model of inflation. In order are the issue of the type of the phase transition, depending on the Higgs mass, or the right form of potential temperature correction terms, see Refs. [33, 34]. In the present theory, however, the vacuum energy is very large, $V^{1/4} \sim \sqrt{M_{Pl}M_H\chi}$, because of the gravity non-minimal coupling, which acts like a negative mass term to induce the phase transition; for a similar view in the context of the SU(5) GUT see Ref. [35]. Therefore, one can expect, for a wide spectrum of Higgs mass values, the temperature corrections to be smaller than the contribution given by the potential Eq. (3). Then, in our cosmological approach, we shall achieve the inflationary stage just before the phase transition takes completely place, that is, when $\chi > 1$. For that reason, when $\chi \sim 0$, the temperature, shifted away due to inflation,

[2] From now on, we shall always consider the dynamics to be dominated by the Higgs terms, in eqs. (18)-(20), instead of the matter density term, from which it is not possible to drive inflation.

shall not play an important role at the Universe dynamics. Temperature considerations shall later be important for entropy and particle production during reheating.

Next, we analyze the initial conditions of our models.

4 INITIAL CONDITIONS AND INFLATION

The initial conditions we have chosen are simply $\dot{a}_o = \dot{\chi}_o = 0$. Equations (18)–(20) must satisfy the following relations:

The size of the initial Universe is if $\epsilon = 1$

$$a_o^2 = \frac{1 + 2\chi_o}{\frac{8\pi G}{3}\rho_o + (1 + \frac{4\pi}{3\alpha})M_H^2\chi_o^2}, \tag{25}$$

its acceleration has the value

$$\frac{\ddot{a}_o}{a_o} = \frac{1}{1+2\chi_o}\Bigg\{ -\frac{4\pi G}{3}\left[1 + \frac{1}{(1+\frac{4\pi}{3\alpha})} + \left(1 - \frac{1}{(1+\frac{4\pi}{3\alpha})}\right)3\nu\right]\rho_o + (1 + \tfrac{4\pi}{3\alpha})M_H^2\chi_o^2 + M_H^2\chi_o\Bigg\} \tag{26}$$

and, for the Higgs field,

$$\ddot{\chi}_o = -M_H^2\chi_o \tag{27}$$

The initial values ρ_o and χ_o as well as M_H are the cosmological parameters to determine the initial conditions of the Universe. The value of M_H fixs the time scale for which the Higgs field breaks down into its ground state. In order to consider the Higgs-terms as the dominant ones (see footnote 2), one must choose the initial matter density $\rho_o < \frac{\chi_o^2}{4\pi}(1 + \frac{4\pi}{3\alpha})M_{Pl}^2 M_H^2$.

The question of the choice of the initial value χ_o is open: for example, within GR for "new inflation" $\chi_o < 0$ [36, 37, 38], whereas for "chaotic inflation" [39] $\chi_o > 0$; from the particle physics point of view one could expect that $\chi_o \sim -1/2$ at the beginning and that it evolves to its broken state $\chi \sim 0$ at the end of the phase transition. But what we know now is just that its actual state is the broken one, and how this has been realized in a cosmological context remains still to be an open question. This extends the theoretical possibilities for cosmological models allowing a mayor window of feasible initial conditions for the scalar field responsible for inflation. For instance, this is of particular interest for the chaotic inflationary cosmology in induced gravity models [16, 17, 18], for which the initial value of the Higgs field must be far from its ground state value, $\phi > v$. Therefore, we are considering both new and chaotic initial conditions, which shall imply different cosmological scenarios.

Scenario (a) ($\chi_o < 0$): From Eq. (25) it follows that if the initial value of the Higgs field is strictly $\chi_o = -1/2$, the Universe possesses a singularity. If the Higgs field sits near to its metastable equilibrium point at the beginning ($\chi_o \gtrsim -1/2$), than χ grows since $\ddot{\chi}_o > 0$, and from Eq. (26) one gets that $\ddot{a}_o < 0$, i.e., a maximum point for a_o; thus at the beginning one has a contraction instead of an expansion. Let us call this *rollover contraction.*

Normally it is argued that in BDT with a constant (or slowly varying) potential producing a finite vacuum energy density, the vacuum energy is dominant and is used to both to expand the Universe and to increase the value of the scalar field. This "shearing" of the vacuum energy to both pursuits is the cause of a moderate power law inflation instead of an exponential one [40]. In this scenario the Universe begins with a contraction, and therefore the same shearing mechanism, moreover here due to the Higgs field, drives a "friction" process for the contraction, due to the varying of $G(\chi)$, making the deflation era always weaker. Furthermore, one can see from Eq. (26) that the cause of the deacceleration is the negative value of χ; then if $\dot{a} < 0$ from Eq. (20) it follows $\ddot{\chi} \sim -\frac{\dot{a}}{a}\dot{\chi} > 0$, which implies an "anti-friction" for χ that tends to reduce the contraction, see also Ref. [41] for a similar view, however, applied to the context of GUT's.

One may wonder if the rollover contraction can be stopped. As long as χ is negative the contraction will not end, but if χ goes to positive values, impulsed by special initial conditions, one could eventually have that the dynamics dominating term, $M_H^2(\chi^2+\chi)$, be positive enough to drive an expansion. But due to the nature of Eq. (20), if χ grows, the term $M_H^2\chi$ will bring it back to negative values and cause an oscillating behavior around zero, its equilibrium state, with an amplitude which is damped with time due to the redshift factor $3H\dot{\chi}$. Therefore, one has to seek special values of χ_o , which will bring χ dynamically from negative values to great enough positive values to end up with sufficient e-folds of inflation. One can understand such a peculiar solution to be reached due to the existence of the "inflation attractor", for which the model of inflation is well behaved, see Ref. [42] and references therein. This feature makes clear that this scenario is not generic for inflation, but depends strongly on special initial conditions; in this sense, this is another type of fine-tuning, which is, indeed, a "quasi-long" standing problem always present by choosing the initial value of the inflaton field in new inflationary scenarios. For instance, in the standard model with a chosen Higgs mass value one finds by numerical integration that obtaining the required amount of inflation implies for χ_o to be that special value given in table 1, see also figures 1(a) and 2(a), but not a very different number than this, otherwise the deflation era does not stop and the Universe evolves to an Einstein Universe with a singularity; one could consider whether GR singularities are an inevitable consequence of particle physics. In doing these computations, it has been also assumed, of course, that during the deflation phase the stress energy of other fields, e.g., radiation or nonrelativistic matter, are smaller than the Higgs one, otherwise an expansion follows.

Scenario (b) ($\chi_o > 0$): One could consider initial conditions whereby the Higgs terms $\chi_o^2 + \chi_o > 0$ dominate the dynamics to have a minimum for a_o, i.e., $\ddot{a}_o > 0$, and to begin on "a right way" with expansion i.e., inflation. That means one should start with a value $\chi_o > 0$ (far from its minimum) positive enough to render sufficient e-folds of inflation. Thus, the "effective" inflaton potential part is similar to the one proposed in the "chaotic" inflationary model [39] because of the form of the potential and the assumed initial Higgs value far from its potential minimum to get eventually the desired inflation, see also [17], but in the present theory, of course, we are regarding a much lower energy scale. Then, both this and chaotic inflation scenarios are generic [5].

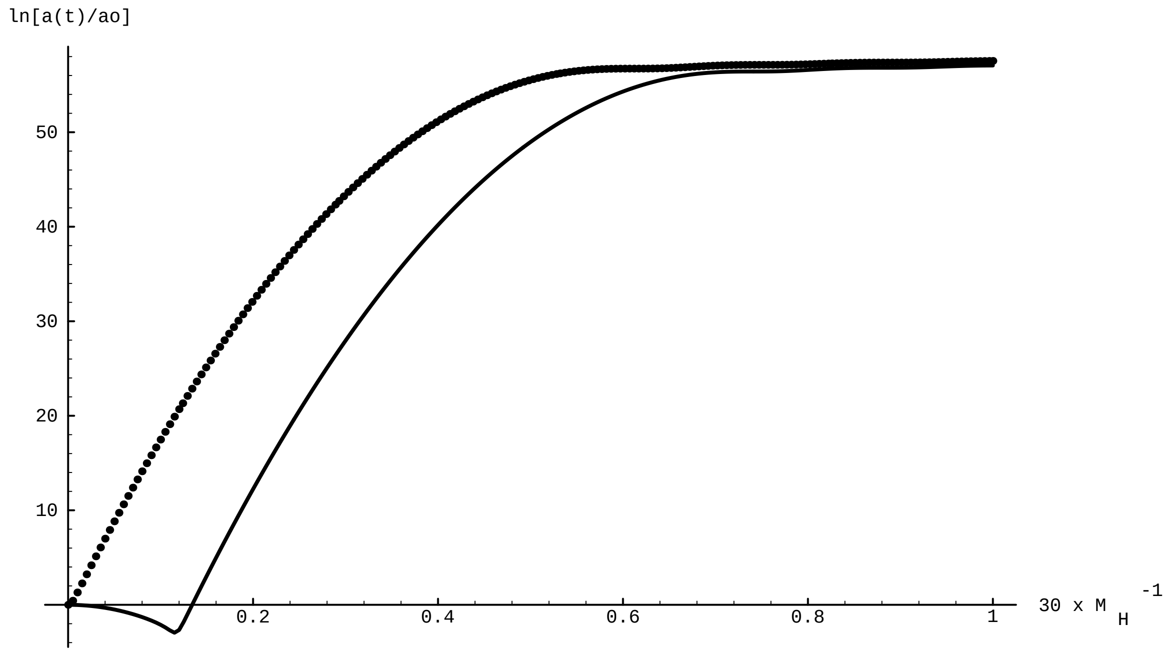

Figure 1: The e-folds of inflation, N, is shown for both inflationary models (a) and (b) in a logarithmic scale. The model (a) begins with a "fast" contraction followed automatically by an inflation if χ_o is that special value given in table 1. The upper curve (scenario (b)) shows the behavior of inflation if $\chi_o \approx 2N_{\min}/3$ (chaotic exponential expansion).

Now let us now see how the dynamics of both scenarios works: The curvature term ϵ/a^2 in (18) can be neglected only after inflation began; during the rollover contraction, scenario (a), it plays an important role. The terms $\frac{\dot{a}}{a}\dot{\chi}$ will be comparable to $8\pi GV(\chi)/3$ untill the high oscillation period ($H < M_H$) starts. For instance, in the chaotic scenario (b), the slow rollover condition $\ddot{\chi} \approx 0$ [3] is valid, which implies $\dot{\chi}/\chi = -M_H^2/3H$, and then from

$$H^2 \approx \frac{1}{1+2\chi}\left[M_H^2\chi^2 - 2H\dot{\chi}\right] \tag{28}$$

(with $\dot{\chi} < 0$) it follows that for $\chi > 2/3$ the Hubble parameter will be dominated by the potential term to have

$$H \approx M_H \frac{\chi}{\sqrt{1+2\chi}} \quad , \tag{29}$$

which for $\chi >> 1$ goes over into $H/M_H \sim \sqrt{\chi/2} >> 1$, giving cause for the slow rollover chaotic dynamics. Indeed, the rollover time is $\tau_{\rm roll} \sim 3H/M_H^2$, i.e.,

$$N = H\tau_{\rm roll} \;\approx\; 3H^2/M_H^2 \;\approx\; 3\frac{\chi^2}{1+2\chi} \quad , \tag{30}$$

[3] this condition is equivalent to both known prerequisites $3H >> \ddot{\phi}/\dot{\phi}$, $3\dot{\phi}/\phi$ of induced gravity.

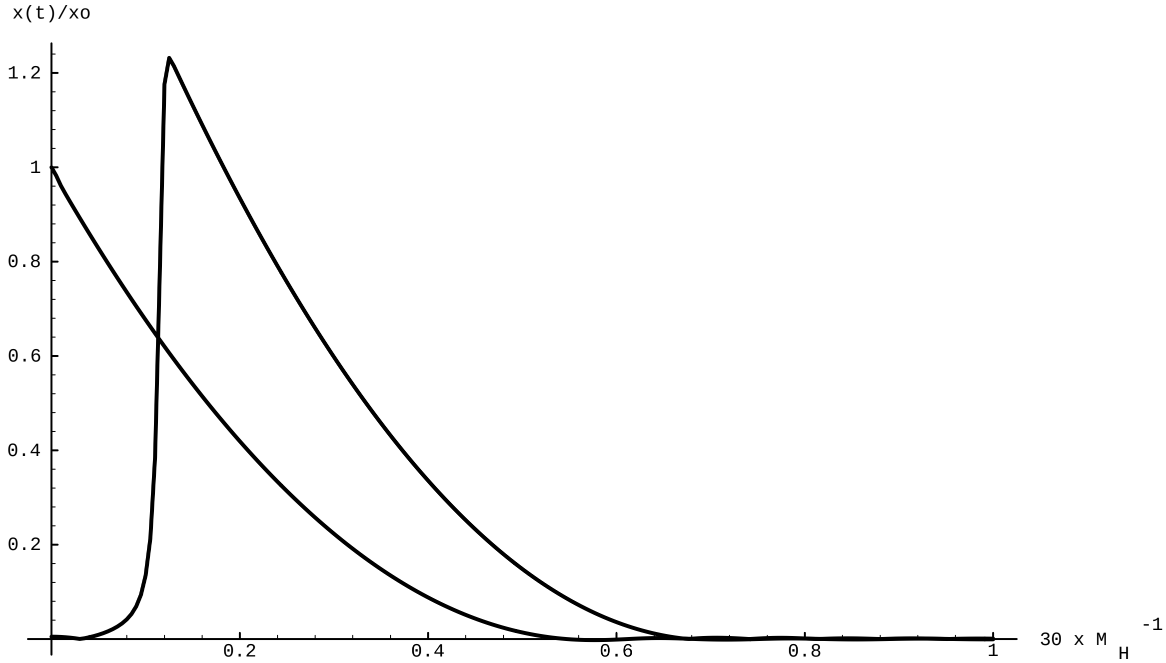

Figure 2: The Higgs field of as a function of time. (a) The Higgs field goes first very fast until it reaches $\chi \gtrsim 2N_{\min}/3$; at that point H evolves faster than χ, to proceed with an inflationary phase. (b) The same as in figure (a) but here with initially $\chi_o \gtrsim 2N_{\min}/3$. The exponential expansion takes place directly (the ordinate axis of figure (a) is divided by 200).

yielding enough e-folds of inflation (N) requires to choose the initial Higgs field value sufficient great, and the slow rollover conditions asures an inflationary stage; for $\chi_o >> 1$ it follows that $\chi_o \gtrsim 2N_{\min}/3$; this value can be checked by numerical integration, see figure 1(b) and 2(b). Note that the required amount of inflation depends only on the initial value of χ_o through $N \gtrsim 3\chi_o/2$, that is, H/M_H does not depend on the energy scale of inflation but on the initial value χ_o; in other words, enough inflation is performed automatically and independently of α as was pointed out in Ref. [17]. Moreover, it is well known that at higher energy scales the amount of e-folds required for successfully inflation is bigger. In this model one can see this as follows: suppose that at the time $t_* \approx 10^l M_H^{-1}$ reheating (RH) takes place and assume that $T_{RH} = \sqrt{M_{Pl} M_H}$ (we use this simple relationship, but a properly account of reheating is developed in Ref. [43]) then one has that

$$N_{\min} = \frac{1}{3}\ln S - \frac{1}{2}\ln\frac{M_{Pl}}{M_H} + \frac{1}{2}\ln\frac{2}{\chi_o} - \frac{2l}{3}\ln 10 \qquad (31)$$

where S is the entropy of the Universe. Therefore the value of $N_{\min}$ should be also greater by increasing the energy scale of inflation. For instance, for $S = 10^{88}$ and $M_H = 10^{-5} M_{Pl}$ (see later discussion) one obtains the $N_{\min} \approx 57$. In table 1 are computed the $N_{\min}$ values for three Higgs mass values.

On the other side if χ_o is negative, the *rollover contraction* phase in scenario (a)

happens, but in this case Eq. (28) indicates $H/M_H \approx |\chi| < 1$, that is, the scale factor evolves slower than the Higgs field; and for special values of χ_o, the Higgs field evolves to values greater or equal than $2N_{\min}/3$ to gain conditions very similar to scenario (b), see figures 1 and 2. In table 1 are given the initial Higgs field values when successfully inflation is achieved in scenario (a) for different Higgs mass values. The very special initial value χ_o depends also only on $N_{\min}$; again because of the fact that at higher energy scales $N_{\min}$ is greater, then χ_o is slightly different for the various Higgs masses in table 1.

Summarizing, for the two possibilities of Universe's models, one has the following: In the chaotic scenario (b), the initial value should be $\chi_o > 2N_{\min}/3$ in order to achieve sufficient e-folds of inflation. And in the scenario (a), only for special initial values of the Higgs field, the Universe undergoes a small contraction which goes over automatically into a sufficiently long inflation period; otherwise, for other initial negatives values of χ_o, the Universe contracts to a singularity. The cosmological model integrated and shown in the figures correponds to $M_H = 10^{14} GeV$ (see discussion later). For the other Higgs mass values, the dynamics is very similar, giving an output resumed in table 1.

At the end of inflation the Higgs field begins to oscillate with a frequency $M_H > H$ and the numerical solution goes smoothly into an oscillation dominated Universe, reaching a normal Friedmann regime [44]. This can be seen as follows: First when $H \gtrsim M_H$ with $H \approx$ const., $\chi \approx e^{-3H/2\ t} cos M_H t$ is valid, later on when $H << M_H$, $H \sim 1/t$ and $\chi \sim 1/t\ cos M_H t$ give rise to $a \sim t^{2/3}$, i.e., a matter dominated Universe with coherent oscillations, which will hold on if the Higgs bosons do not decay; in figures 3 and 4 the behavior of the scale factor and the Higgs field is shown until the time $100 M_H^{-1}$; the numerics fit very well the "dark" matter dominated solutions. Let us consider this possibility more in detail: then, the average over one oscillation of the absolute value of the effective energy density of these oscillations, $\rho_\chi \gtrsim V(\chi) = \frac{3}{4\pi} M_{Pl}^2 M_H^2 \chi^2$, is such that

$$\frac{\rho_\chi}{\rho_{\chi_{\rm osc}}} = \left(\frac{t_{\rm osc}}{t}\right)^2 \tag{32}$$

where $t_{\rm osc}$ is the time when the rapid oscillation regime begins. From Eq. (32) one can compute the present (labeled with a subindex n) energy density of these oscillations if they were to exist. Then,

$$\rho_{\chi_n} \gtrsim \frac{3}{4\pi} M_{Pl}^2 M_H^2 \chi_{\rm osc}^2 \left(\frac{t_{\rm osc}}{t_n}\right)^2 . \tag{33}$$

From the figures it is evident that $t_{\rm osc} = 20 M_H^{-1}$ and $\chi_{\rm osc} = 10^{-2}$, and $t_n \sim 10^{17}{\rm s} = 1.5 \times 10^{41} GeV^{-1}$ implying, for all Higgs mass values chosen in table 1, that $\rho_{\chi_n} \gtrsim 3 \times 10^{-47} GeV^4$, i.e., the Higgs oscillations could solve the missing mass problem of cosmology, implying the existence of cold dark matter, since after some time as the Universe expands the Higgs particles will have a very slow momentum owing to their big mass. Furthermore, if some amount of Higgs oscillations decays into relativistic particles with $\rho \sim 1/a^4(t)$, they can dominate the dynamics of the Universe only for a time era, until the density of the remanent oscillations (if they are still there), decreasing as $1/a^3(t)$, govern again the scale factor evolution, giving place also in this case to a dark matter dominated Universe, even if the remanents strongly interact with each other [43].

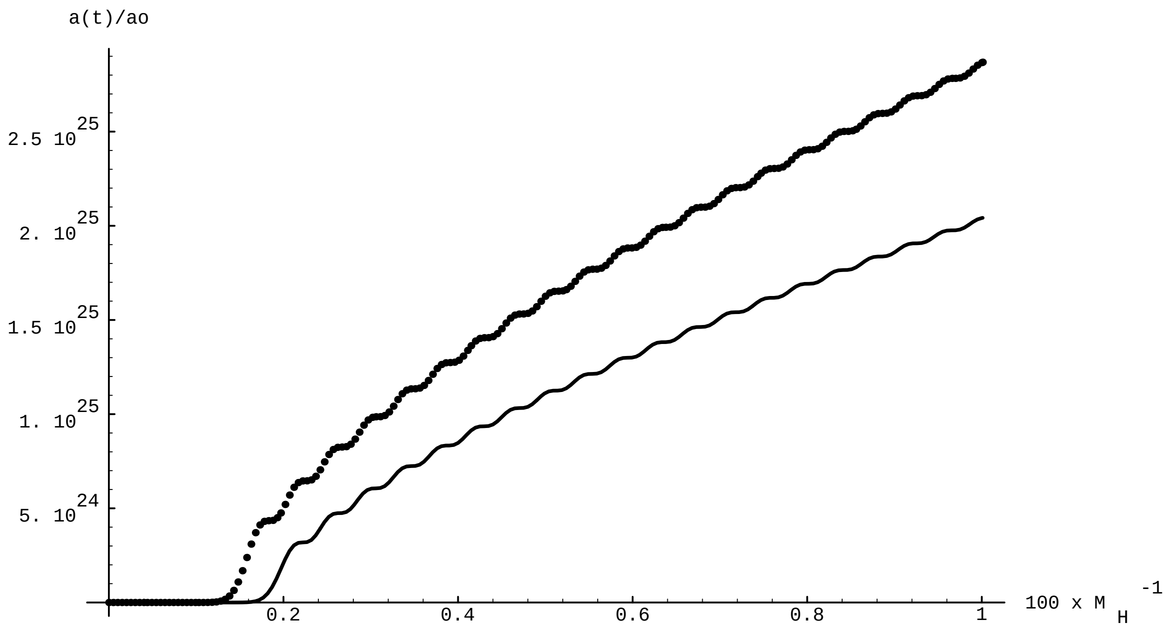

Figure 3: Again the scale factor evolution as in figure 1, but now till $t = 10^2 M_H^{-1}$. One notes that the inflation time is approximately $t = 20 \times M_H^{-1}$, later on, the Universe is "dark" matter dominated, perhaps until today, if reheating didn't take away the coherent Higgs oscillations. It can be seen the track imprinted by the Higgs coherent oscillations in the scale factor evolution at that time scale; later on, this influence will be imperceptible.

There is, however, a very important problem: if one tries to explain the today observed baryonic mass of the Universe, given by $M(t) \approx M(1+2\chi)^{\frac{1}{2}(1-3\nu)}/a^{3\nu}$, one has that after inflation it is too small unless the perfect fluid behaves like dust particles ($\nu = 0$); moreover, the temperature at that time should also be too small. For solving these problems one has to assume that some amount of the Higgs oscillations decay into baryons and leptons. At the time around t_* the Higgs field should decay into other particles with a decay width Γ_H to give place to a normal matter or radiation Universe expansion, producing the reheating of the Universe [45, 46, 47]. If reheating takes place, the still remaining energy of the scalar field at t_* is converted into its decay products. This would mean that the cosmological "function" disappears to give then rise to the known matter of the Universe. But if coherent oscillations still stand, they are the remanents of that cosmological "function", which is at the present however invisible to us in the form of cold dark matter, moreover, the Higgs particle does not interact with the rest of the particles but only gravitationally, therefore, the Higgs oscillations don't change baryogenesis and/or nucleosynthesis processes. Now suppose that the oscillations really did decay. Mathematically, the way of stopping the oscillations or to force the decay is to introduce a term $\Gamma_H \dot{\chi}$ in Eq. (20). The Universe should then reheat up to the temperature $T_{RH} \approx \sqrt{M_{Pl}\Gamma_H}$ where Γ_H depends, of course, on the decay products, see however Ref. [43] for a more realistic reheating scenario.

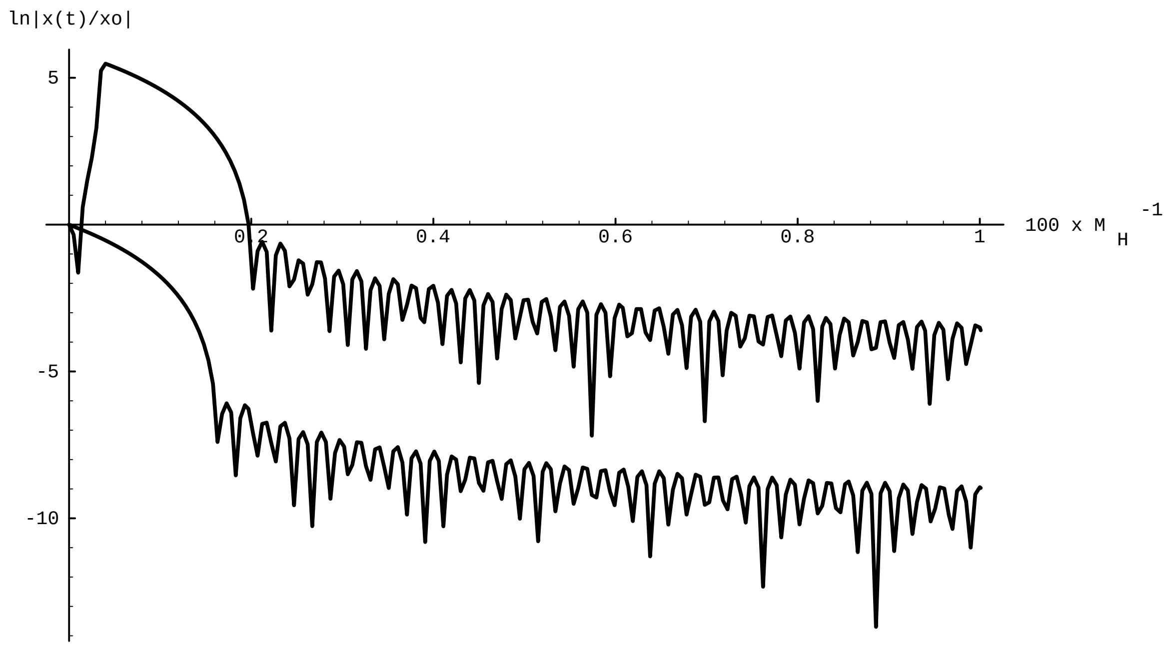

Figure 4: The evolution of the Higgs oscillations is shown in logarithmic scale during and after inflation. In scenario (a) the Higgs field jumps from very small values to $2N_{\min}/3$ to achieve inflation, later it begins to oscillate. In scenario (b), the Higgs field disminishes until it begins to oscillate.

For example, if the coherent oscillations decay into two light fermions, it is valid that $\Gamma_H \approx g^2 M_H$; for the reheating this would mean that $T_{RH} \approx g\sqrt{M_H M_{Pl}}$ which should be enough for non-GUT baryogenesis to occur [48], for the given Higgs mass values of table 1. But one should be aware of the production of gravitational radiation as a decay output of the oscillations [29]; however, it could be also possible that other decay channels are important, since the symmetry-breaking takes here place at a much more smaller energy scale than the Planck one.

The contrast of density perturbations $\delta\rho/\rho$ can be considered in scenario (b), or in scenario (a) when χ has evolved to its maximal value to have very similar slow rollover conditions as in (b). Then one has [17, 49]

$$\left.\frac{\delta\rho}{\rho}\right|_{t_1} \approx \frac{1}{\sqrt{1+\frac{3\alpha}{4\pi}}} H \left.\frac{\delta\chi}{\dot{\chi}}\right|_{t_1} = \sqrt{\frac{3}{\pi\alpha}} \left(1+\frac{4\pi}{3\alpha}\right) \frac{M_H}{v} \left.\frac{\chi^2}{1+2\chi}\right|_{t_1} \tag{34}$$

where t_1 is the time when the fluctuations of the scalar field leave H^{-1} during inflation.

Table 1: The Higgs field initial values yielding sufficient inflation for three Higgs masses in both new and chaotic scenarios. We have overall taken $\dot{\chi}_o = 0$.

Higgs mass M_H	$\mathrm{e-folds}$ $N_{\min}$	new inflation χ_o	chaotic inflation $2N_{\min}/3$
$10^{14}GeV$	57	-0.155088	38
$10^{2}GeV$	44	-0.155061	29
$10^{-6}eV$	24	-0.155050	16

At that time, one finds that

$$\frac{\delta\rho}{\rho}\bigg|_{t_1} \approx \frac{2}{\sqrt{3}}\frac{\sqrt{\lambda}}{\alpha}\frac{\chi^2}{1+2\chi}\bigg|_{t_1} \approx \sqrt{\frac{1}{6\pi}}\frac{M_H}{M_{Pl}}\ N(t_1) \ \approx\ 10\frac{M_H}{M_{Pl}} < 10^{-4} - 10^{-5}\ , \tag{35}$$

where we have used $\left(1+\frac{4\pi}{3\alpha}\right) \approx 1$ and we recall that $N(t_1) = \frac{3\chi^2}{1+2\chi}\Big|_{t_1}$ from Eq. (30), which can be numerically checked from figure 4. In order to have an acceptable value of $\delta\rho/\rho \approx 10^{-5}$ one is forced to choose $M_H < 10^{-5} - 10^{-6} M_{Pl}$. In that way the magnitud of density perturbations can give rise to the observed astronomic structures, corresponding approximately to $N(t_1) = 50$ e-folds before inflation ends. The accomplishment of the right density contrast at this energy scale determines a very large value for $\lambda >> 1$, making a tide interaction at the outset of inflation; this huge value for λ brings the energy scale of inflation to be approximately as great as the GUT's inflationary scenarios; in the induced gravity model considered in Ref. [18] Eq. (35) also holds, that is, this equation is a caracteristic of all induced gravity models, as was also pointed out in Refs. [17, 49]. If one assumes that $\lambda \approx \alpha$, the Higgs mass becomes of the order of magnitud of the electroweak scale ($G_F^{-1/2}$), then, this theory is equivalent to a massive Yang-Mills theory, which is in agreement with the present experiments even though it is non renormalizable, because the cut-off dependence is only logarithmic [21]. On the other hand, if $\lambda \sim 1$ implies $M_H \approx 10^{-6}eV$ and hence inflation should be realized at approx. $t \sim 10^{-9}$s. This could also be possible, but then $\delta\rho/\rho$ is extremely small, leaving the structure formation problem aside from inflation. In the latter case, the reheating temperature is about $10^2 GeV$, *i.e.*, on the limit for non-GUT baryogenesis to occur [48].

The spectral index of the scalar perturbations, n_s, serves has a test for models of

the very early Universe, indenpendently of the magnitud of the perturbations and can be calculated, using the slow roll approximation up to second order [50, 42], however, for $\alpha >> 1$, one can just take the first order to be sufficiently accurate [51]:

$$n_s = 1 - \frac{2\alpha}{N\ \alpha + \pi} \approx 1 - \frac{2}{N} \ , \tag{36}$$

for $N = 50$, it implies $n_s \approx 0.96$ in accordance with the recent COBE DMR results [8].

The perturbations on the microwave background temperature are also well fitted. The gravitational wave perturbations considered normally must also be very small [11],

$$h_{GW} \approx \frac{H}{M_{Pl}} \approx \frac{M_H}{M_{Pl}} \sqrt{\frac{\chi}{2}} \bigg|_{t_1} < 10^{-5} \ , \tag{37}$$

for the all above mentioned Higgs mass values.

5 CONCLUSIONS

We have presented the induced gravity model coupled to the standard model of particle physics, where the cosmological inflaton is precisely the SU(2) isovectorial Higgs field. As a consequence of this, one has some new features for both particle physics and cosmology. It was shown that the combination of the fundamental masses of the theory, due to the non-minimal gravity coupling, in a natural way fixes α to be $\sim 10^{33}$ and the Higgs mass to be $\sqrt{\frac{4\pi}{3\alpha}}$ less than that in the standard SU(2) theory. Indeed, the excited Higgs decouples from the other particles and interacts just by means of the very weak gravitational field contained in the space-time covariant derivative. Also, because of the non-minimal coupling the vacuum energy responsible for inflation is $V^{1/4} \sim \sqrt{M_{Pl} M_H \chi}$, bringing the energy scale of inflation equal to that of the Higgs mass, by means of Eq. (29). The cosmological equations (18-21) present for $\chi_o < 0$ a *rollover contraction* era in scenario (a), whereby only for special initial values the model can evolve to its "inflaton attractor" giving rise, after all, to a chaotic scenario, where inflation takes place by the virtue of a normal rollover approximation.

After inflation, the universe is oscillation dominated, and without its total decay one could explain the missing mass problem of cosmology given today in the form of cold dark matter.

As a matter of fact, the cosmological model cannot explain by itself the today observed baryon mass of the universe, for which one is forced to look for a reheating scenario after inflation. A carefully treatment of it is not developed here, but elsewhere [43]; nevertheless it was point out the reheating Temperature should be enough for non-GUT baryogenesis to occur. However, the question whether to much gravitational radiation is generated to eventually spoil a normal nucleosynthesis procedure remains open at this energy scale.

The right amplitude of scalar and tensor density perturbations required to explain the seeds of galaxy formation imposes very great values to the Higgs mass, $M_H < 10^{-5}$-$-10^{-6} M_{Pl}$, otherwise, inflation at lower energy scales does not account for solving that problem. At any energy scale, induced gravity models predict a value of the spectral index, $n_s \sim 1$, according with the recent observations, see Refs. [51, 8].

As a final comment we would like to point out that the induced gravity model in the SU(5) theory seems to be better accomplished because, in that case, the ratio of the Higgs to Planck mass is in a natural way of the order of 10^{-5} [18], achieving right perturbation amplitudes. Contrary to that case, the present SU(2) Higgs gravity inflationary model requires unnatural big Higgs mass values in order to render a successfully cosmology. This is, somehow, the price paid in matching gravity to a very low energy scale; it reminds us once more of a quasi-long standing problem of inflation: whereas cosmology is happy, particle physics is infelicitous, or inversely.

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